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Mark Scheme (Results)

Summer 2015

Pearson Edexcel GCE in
Mechanics 4 (6680/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

<u>'M' marks</u>
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

<u>'A' marks</u>
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

<u>'B' marks</u>
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

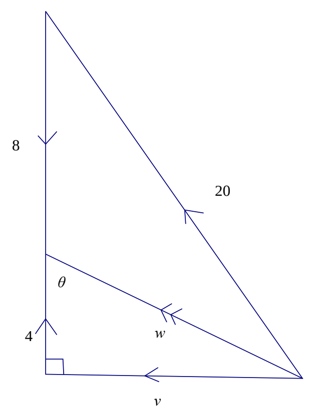
  RHS, LHS Right hand side, left hand side.

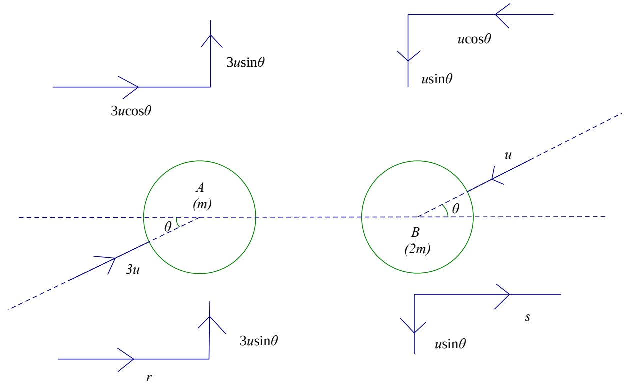
June 2015
6680 Mechanics 4
Mark Scheme

Question Number	Scheme	Marks	Notes
1	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Find position vector of one particle relative to the other . $\mathbf{r}_P = \begin{pmatrix} 16+t \\ -12+2t \end{pmatrix}$, $\mathbf{r}_Q = \begin{pmatrix} -5+2t \\ 4+t \end{pmatrix}$
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	$d^2 = (21-t)^2 + (-16+t)^2$	M1	Pythagoras
	$\frac{\mathrm{d}}{\mathrm{d}t} d^2 = -2(21-t) + 2(-16+t)\ (= -74+4t)$	M1	Differentiate d or d^2 wrt t
			Set derivative = 0 and solve for t
	Min when $t = 18.5(\mathrm{s})$	A1	
	Relative position $\begin{pmatrix} 2.5 \\ 2.5 \end{pmatrix}$, distance $\sqrt{2.5^2 + 2.5^2}$ (m)	M1	Substitute their t to find d
	$= \sqrt{\frac{25}{2}} = 3.54$ (m)	A1	
		[7]	
	See over for alternatives.		

Question Number	Scheme	Marks	Notes
alt1	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Position of P relative to Q
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	$d^2 = (21-t)^2 + (-16+t)^2 \; (= 2t^2 - 74t + 697)$	M1	Use Pythagoras to express d^2 as a quadratic in t
		M1	Complete the square
	$2(t-18.5)^2(-684.5+697)$	A1	Correct as far as $2(t-18.5)^2 + ...$
	Min $d^2 = 697 - 684.5$	M1	Use completed square to find minimum value for their expression
	Min. $d = \sqrt{697-684.5} = \sqrt{12.5}$	A1	
alt2	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Position of P relative to Q
	$= \begin{pmatrix} 21-t \\ -16+t \end{pmatrix}$	A1	Accept +/-
	Relative velocity $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	M1	
	$: \begin{pmatrix} 21-t \\ -16+t \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -(21-t) + (-16+t) = 0,$	M1	Set scalar product of relative position and relative velocity = 0 and solve for t.
	$t = 18.5\,(\text{s})$	A1	
	Relative position $\begin{pmatrix} 2.5 \\ 2.5 \end{pmatrix}$, distance $\sqrt{2.5^2 + 2.5^2}$ (m)	M1	Substitute their t to find d
	$= \sqrt{\frac{25}{2}} = 3.54$ (m)	A1	
	See over for alternative		

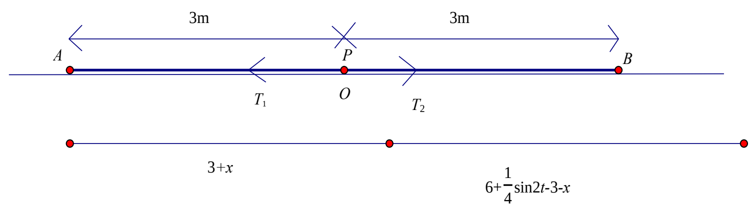
Question Number	Scheme	Marks	Notes
alt 3	$\mathbf{r}_P - \mathbf{r}_Q$	M1	Initial position of P relative to Q
	$= \begin{pmatrix} 21 \\ -16 \end{pmatrix}$	A1	Accept +/-
	Relative velocity $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$	M1	
		M1	Use scalar product to find $\cos\theta$
	$\cos\theta = \frac{-37}{\sqrt{2}\sqrt{697}}$ (-0.998....)	A1	Accept +/-
	$d = PQ\sin\theta$	M1	Use trig to find distance
	$= \sqrt{697} \times \sqrt{1 - \frac{37^2}{2 \times 697}} = \frac{5}{\sqrt{2}} \approx 3.54$	A1	
		[7]	

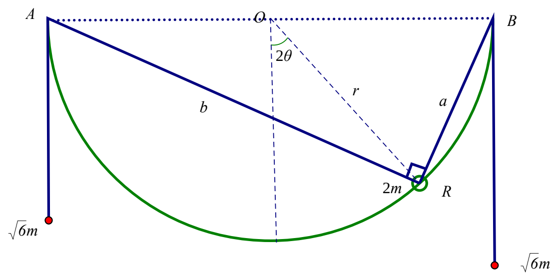
Question Number	Scheme	Marks	Notes
2		B1	Either triangle of velocities
		M1	Two triangles combined using their common velocity
		A1	Correct diagram seen or implied
	Correct method to obtain one of v, w, θ	DM1	($v = 16, w = 16.5, \theta = 76°$) Dependent on the previous M1
	speed is 16.5(km h^{-1})	A1	$4\sqrt{17}$
	Direction S 76° E or equivalent	A1	104° or equivalent
		[6]	
Alt	Velocity of wind = w		
	$w = -v\mathbf{i} + 4\mathbf{j}$	B1	one correct equation
	$w = a\mathbf{i} + b\mathbf{j} - 8\mathbf{j}$ $\quad a^2 + b^2 = 400$	M1	2nd equation and compare coefficients
	coeff $\mathbf{j}$: $4 = b - 8$ $\quad b = 12$	A1	2 correct eqns
	$\mathbf{i}$: $-v = a$		
	$a^2 + 144 = 400 \Rightarrow a = -16 \quad (v > 0)$	DM1	Dependent on the previous M1
	$\lvert w \rvert = \sqrt{4^2 + 16^2} = 4\sqrt{17}$	A1	
	Bearing 104°	A1	or equivalent

Question Number	Scheme	Marks	Notes
3		B1	After collision $u\sin\theta$ and $3u\sin\theta$ perpendicular to l of c Seen or implied
	CLM : $r+2s=3u\cos\theta-2u\cos\theta(=u\cos\theta)$	M1	Requires all four terms but condone sign errors and consistent sin/cos confusion.. Must be dimensionally consistent
		A1	Correct unsimplified equation
	Impact: $s-r=e\times 4u\cos\theta\left(=\dfrac{u\cos\theta}{2}\right)$	M1	Must be the right way round, but condone sign errors and consistent sin/cos confusion
		A1	Correct unsimplified equation. Signs consistent with CLM equation.
	$\Rightarrow r=0, s=\dfrac{u\cos\theta}{2}$	DM1	Solve the simultaneous equations to find the horizontal components of velocities. Dependent on the two preceding M marks
		A1	Both correct
	After the collision: $(3u\sin\theta)^2+r^2=4\left((u\sin\theta)^2+s^2\right)$	M1	Use $v_A=2v_B$. Condone 2 in place of 4.
		A1ft	Correct unsimplified equation (in r and s)
	$9u^2\sin^2\theta=4u^2\sin^2\theta+4.\dfrac{u^2}{4}\cos^2\theta$	A1	Obtain an equation in θ (correct only)
	$\tan^2\theta=\dfrac{1}{5}$, $\theta=24.1(°)$ (0.421 radians)	DM1	Solve for θ. Dependent on the previous M1
		A1	Correct to 3 sf or better

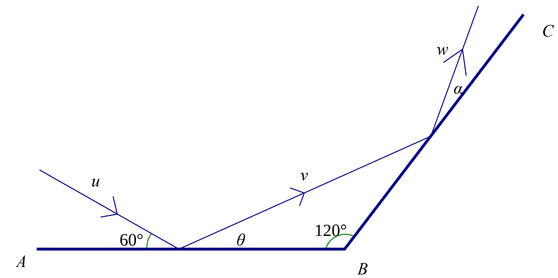
Question Number	Scheme	Marks	Notes
3 alt	For those who prefer everything with trig:		
	$v_A \sin\alpha = 3u\sin\theta$, $v_B \sin\beta = u\sin\theta$	B1	Perpendicular to the l.o.c.
	$m.3u\cos\theta - 2m.u\cos\theta = mv_A\cos\alpha + 2mv_B\cos\beta$	M1	CLM
	$(u\cos\theta = v_A\cos\alpha + 2v_B\cos\beta)$	A1	
	$\frac{1}{8}\times(3u\cos\theta + u\cos\theta) = v_B\cos\beta - v_A\cos\alpha$	M1	Impact law
	$\left(\frac{u}{2}\cos\theta = v_B\cos\beta - v_A\cos\alpha\right)$	A 1	
	$\frac{u}{2}\cos\theta = v_B\cos\beta$, $0 = v_A\cos\alpha\,(\Rightarrow \sin\alpha = 1)$	DM1	Simultaneous equations
		A1	
	$v_A\sin\alpha = v_A = 2v_B = 3u\sin\theta$	M1	Use $v_A = 2v_B$ to find β
	$v_B\sin\beta = u\sin\theta \Rightarrow \frac{3u\sin\theta}{2}\sin\beta = u\sin\theta$	A1	Equation without v_A and v_B
	$\sin\beta = \frac{2}{3}$	A1	
	$2v_B = 3u\sin\theta$ & $\frac{u}{2}\cos\theta = v_B\cos\beta$ $\Rightarrow 6\tan\theta = \frac{2}{\cos\beta}\left(= 2\times\frac{3}{\sqrt{5}}\right)$	M1	Solve for θ
	$\tan\theta = \frac{1}{\sqrt{5}}$, $\theta = 24.1(°)$ (0.421 radians)	A1	
		[12]	

Question Number	Scheme	Marks	Notes
4a	Equation of motion: $900a = \dfrac{22500}{v} - 25v$	M1	Requires all three terms. Condone sign errors
		A1	Correct unsimplified equation
	$a = \dfrac{\dfrac{22500}{v} - 25v}{900} = \dfrac{900 - v^2}{36v}$	A1 [3]	Obtain ****Given answer**** with no errors seen
4b	$\dfrac{dv}{dt} = \dfrac{900 - v^2}{36v}$	B1	Differential equation in v and t
	$\int \dfrac{36v}{900 - v^2} dv = \int 1 dt$,	M1	Separate & integrate to obtain a solution involving a log. function
	$t = -18\ln(900 - v^2)(+C)$	A1	
	: $T = -18\ln 500 + 18\ln 800 = 18\ln\dfrac{8}{5}$	DM1	Use limits correctly Dependent on previous M1
		A1	Obtain ****Given answer**** with no errors seen
		[5]	
4c	$\dfrac{900 - v^2}{36v} = v\dfrac{dv}{dx}$	B1	Differential equation in v and x
	$\int \dfrac{v^2}{900 - v^2} dv = \int \dfrac{1}{36} dx$	M1	Separate variables
	$= \int \dfrac{900}{900 - v^2} - 1 dv = \left(\int \dfrac{900}{60}\left(\dfrac{1}{30 - v} + \dfrac{1}{30 + v}\right) - 1 dv\right)$	M1	Split to the form $\dfrac{A}{900 - v^2} + B$ and integrate
	$15\ln\left\lvert\dfrac{30 + v}{30 - v}\right\rvert - v = \dfrac{x}{36}(+C)$	A1	
	$15\ln\left(\dfrac{50}{10} \times \dfrac{20}{40}\right) - (20 - 10) = \dfrac{x}{36}$	M1	Use limits and solve for x
	$x = 135$ (m) $\quad (540\ln 2.5 - 360)$	A1	Accept exact answer of the form $a\ln b - c$
		[6]	
		(14)	

Question Number	Scheme	Marks	Notes
5a			Extension in AP : $2+x$, Extension in BP : $3+\frac{1}{4}\sin 2t - x - 1$
	$T_1 = \frac{12(2+x)}{1}$	B1	Force towards A
	$T_2 = 12\left(2+\frac{1}{4}\sin 2t - x\right)$	B1	Force towards B
	$1.5\frac{d^2x}{dt^2} = T_2 - T_1 (= 3\sin 2t - 24x)$	M1	Form equation of motion of P. Requires derivative and both tensions, but condone sign errors. Allow with a for $\ddot{x}$
		A1	Correct equation in x and t
	$\frac{d^2x}{dt^2} + 16x = 2\sin 2t$	A1	Obtain *****given answer***** with no errors seen.
		[5]	
5b	$t=0, x=0 \quad \Rightarrow C=0$	B1	
	$t=0, \dot{x}=0=(-4C\sin 4t)+4D\cos 4t+\frac{1}{3}\cos 2t$	M1	SR: M1A1 is available if C is not found or C is incorrect.
	$D=-\frac{1}{12}$	A1	
	$\dot{x}=0 \Rightarrow \cos 4t = \cos 2t$	M1	At rest: set their $\dot{x}=0$
	$2\cos^2 2t - 1 = \cos 2t$		
	$\cos 2t = 1, -\frac{1}{2} \quad 2t = \frac{2\pi}{3}, \quad t = \frac{\pi}{3} \quad (1.05)$	A1	Not $\frac{1}{2}\cos^{-1}\left(\frac{-1}{2}\right)$?
		[5]	

Question Number	Scheme	Marks	Notes
6a			
	GPE of the ring: $-2mgr\cos 2\theta$	B1	Allow with $+2kmgr$
	GPE of suspended particles: $-\sqrt{6}mg(L_1-a)-\sqrt{6}mg(L_2-b)$	M1	Expression of the correct structure involving their L_1, L_2, a and b
	$a=2r\sin(45-\theta)=\frac{2r}{\sqrt{2}}(\cos\theta-\sin\theta)$	A1	Correct expression for BR in terms of r and θ Any equivalent e.g. $r\sqrt{2(1-\sin 2\theta)}$
	$b=2r\cos(45-\theta)=\frac{2r}{\sqrt{2}}(\cos\theta+\sin\theta)$	A1	Correct expression for AR in terms of r and θ Any equivalent e.g. $r\sqrt{2(1+\sin 2\theta)}$
	GPE of system: $-\sqrt{6}mg(L_1-a)-\sqrt{6}mg(L_2-b)-2mgr\cos 2\theta$	DM1	Add the three components. Dependent on the previous M
	$=2\times\frac{2r}{\sqrt{2}}\cos\theta\times\sqrt{6}mg-2mgr\cos 2\theta$ + constant		
	$=2mgr\left(2\sqrt{3}\cos\theta-\cos 2\theta\right)+$ constant	A1	Simplify to the **given answer**
		[6]	

Question Number	Scheme	Marks	Notes
6b	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = -4\sqrt{3}\ mgr\sin\theta + 4mgr\sin 2\theta$	M1	Differentiate
	In equilibrium: $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0 = 4mgr\sin\theta\left(-\sqrt{3} + 2\cos\theta\right)$	M1	Set $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$ and solve for θ
	$\theta = \pm\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \pm\frac{\pi}{6}\left(= \pm 0.52\right)$	A1	
	or $\theta = 0$	B1	
		[4]	
6c	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -4\sqrt{3}mgr\cos\theta + 8mgr\left(\cos^2\theta - \sin^2\theta\right)$	M1	Second derivative - needs to be the full expression.
	$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = mgr\left(-4\sqrt{3}\times\frac{\sqrt{3}}{2} + 8\left(\frac{3}{4} - \frac{1}{4}\right)\right) = -2mgr < 0$	DM1	Substitute $\theta = \frac{\pi}{6}$ Dependent on the previous M1
	So equilibrium is unstable	A1	No errors seen
		[3]	
		(13)	
	Accept equivalent methods for determining max/min.		

Question Number	Scheme	Marks	Notes
7a	Resolve parallel to barrier - condone sin/cos confusion	M1	
	$u\cos 60 = v\cos\theta$	A1	
	Resolve perpendicular to the barrier - condone consistent sin/cos confusion. Use e correctly	M1	
	$eu\sin 60 = v\sin\theta$	A1	
	$v^2 = u^2\cos^2 60 + e^2u^2\sin^2 60 = \frac{u^2}{4} + \frac{3u^2}{16} = \frac{7u^2}{16}$	M1	Eliminate θ and solve for v.
	$v = \frac{\sqrt{7}}{4}u$	A1	Obtain given answer correctly with no errors seen
		[6]	
7b	Angle of approach with BC = 19.1°	B1	
	$v\cos 19.1 = w\cos\phi$	M1	Components parallel to BC condone sin/cos confusion
	$\frac{1}{2}v\sin 19.1 = w\sin\phi$	M1	Components perpendicular to BC condone consistent sin/cos confusion Use e correctly
		A1	Equations correct for their 19.1
	Form equation in v and ϕ	M1	Square and add or divide to find $\tan\phi$
	$w^2 = v^2\left(\frac{1}{4}\sin^2 19.1 + \cos^2 19.1\right)$	A1	$(\phi = 9.83°)$
	$0.634u$	A1	

Question Number	Scheme	Marks	Notes
7balt	$\tan\theta = \frac{1}{2}\tan 60$	B1	
	$\tan\alpha = \frac{1}{2}\tan(60-\theta)$ $\left(=\frac{1}{2}\left(\frac{\sqrt{3}-\frac{1}{2}\sqrt{3}}{1+\sqrt{3}.\frac{1}{2}\sqrt{3}}\right)=\frac{\sqrt{3}}{10}\right)$	M1	
		A1	
	$v\cos(60-\theta) = w\cos\alpha$	M1	
	$v\left(\frac{1}{2}.\frac{2}{\sqrt{7}}+\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{\sqrt{7}}\right) = w\frac{10}{\sqrt{103}}\left(=v\frac{5}{2\sqrt{7}}\right)$	M1	
		A1	
	$w = \frac{\sqrt{103}}{4\sqrt{7}}v = \frac{\sqrt{103}}{4\sqrt{7}}.\frac{\sqrt{7}}{4}u = \frac{\sqrt{103}}{16}u$ $(0.634u)$	A1	
		[7]	

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